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Supporting Algebraic Understanding: A Comparative Study of Secondary School Mathematics Texts

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Abstract

This study examines how algebra is conceptualised within the planned curriculum in England and China, focusing on how teaching materials support the development of algebraic thinking skills. The analysis drew on current teaching resources (Chinese textbooks and English lesson slides) in two nations. The findings revealed distinct pedagogical orientations toward fostering algebraic thinking. In England, the development of abstract thinking is supported through connections to real-world contexts and the use of multiple representations. In contrast, Chinese materials adopt a more formal and abstract approach, emphasising symbolic manipulation. Overall, the findings point to fundamentally different pedagogical traditions in algebraic notation instruction and distinctive approaches to linking arithmetic and algebraic thinking.

Keywords

Algebraic notation; Comparative; Planned curriculum; Textbook analysis; Understanding

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1. Introduction

Towards the end of the 20th century, there was a flourishing of research on textbooks, hardly surprising given the numbers being used around the world. In 1990, Newton noted that in the UK alone, about 30 million school textbooks would be sold to support teaching, learning and curricula. In his book, Teaching with Text, he quoted: "The text is not simply a teaching aid, like a globe, or the models used in geometry lessons. It can be a self-contained teaching unit [1]. He goes on to describe how the textbook, used in a variety of contexts, behaves like a surrogate teacher. Despite the rise of the internet and teachers' access to online materials to support their planning and curriculum delivery, textbooks still seem to have some popularity today. In mathematics education, textbooks are still frequently used to support curriculum planning and pedagogic activity [2]. Some studies have focused on the mathematical content and curricular activities, whereas others have also explored mathematical processing of the problems presented, for example problem solving and mathematical reasoning [3–5].

When such studies make international comparisons between East Asian countries and those in Europe and the USA, the findings can contrast greatly. For example, they found textbooks in East Asia good at conveying ideas but less motivating [6]. Yet in England and the USA, the opposite seemed to hold true. Western textbooks were good at explicit relevance, tying experiences to real life, but less effective at supporting conceptual understanding and the mental connections between the concepts and the real world. Moreover, generic cultural features were noted in some textbooks.

The picture is a complex one, therefore this study aims to investigate the algebra tasks and activities in the textbooks used in two very different education settings (China and England). Furthermore, the literature review revealed a lack of published research on how algebraic notation is introduced in textbooks from East Asian and Western contexts. Accordingly, this study also explores how algebraic notation and understanding are represented in textbooks from England and China.

2. Algebra and algebraic notation

Algebra occupies a central position in mathematics education. Traditionally, algebra has been introduced at the secondary level, typically between the ages of 13 and 15, while arithmetic and number sense have been the focus of primary education. Cai and Knuth argue that this historical divide may explain why many students encounter persistent difficulties when first engaging with algebra [7]. Linchevski and Livneh identified two broad perspectives on the nature of algebra that help clarify this issue [8]. The first regards algebra as a new, formal system, a symbolic language in which letters represent unknowns. As Soneira notes, this symbolic language differs fundamentally from everyday natural language in its structure and meaning, often leading to confusion when students must maintain consistent symbol meanings in contexts that, in ordinary language, would allow for change [9]. In contrast, a second view presents algebra as a natural outgrowth of arithmetic, arising through processes of generalisation and abstraction from numerical and quantitative reasoning [10].

From this standpoint, algebraic understanding develops gradually as learners build on their arithmetic experiences to express and reason about relationships in increasingly general forms. Hackenberg and Lee highlight that algebra involves both recognising general patterns within arithmetic and learning to reason with symbolic notation that stands in place of specific quantities [11].

Drawing on this developmental perspective, as well as the foundational work of Kaput, the present study views the emergence of algebraic reasoning as a continuous process leading from generalised arithmetic thinking to formal symbolic representation [12]. This view provides a conceptual basis for exploring how students encounter and interpret algebraic notation, and how such notation functions as a bridge between intuitive arithmetic reasoning and formal algebraic expression.

3. Research questions

This study was guided by two key research questions:

(1) In what ways do the selected textbooks conceptualise and present the teaching and learning of algebra?

(2) To what extent do differences emerge in the ways examples are used to support algebraic thinking?

4. Methods and materials

4.1. Selection of textbooks and materials

Hodgen et al. indicated that the percentage of lessons where textbooks are the principal basis for instruction is 43% in the UK compared to the international average of 60% [13]. In England, commercial packages with ready-made slides, such as White Rose Maths and Mathematics Mastery, are commonly purchased by schools. Kangaroo Maths was selected as a representative example of widely used online materials in English secondary schools. In China, following the introduction of the Mathematics Curriculum Standards for Compulsory Education (2022 version), textbooks were also revised accordingly. This study analysed the latest edition of Mathematics for Lower Secondary School (Grade 7), published by Zhejiang Education Press. The analysis focused on worked examples provided for learners to support their understanding of algebraic notation in England and China. In England, these often present a problem, work through steps to solve it, and provide the “correct” solution.

In this study, the analysis focuses on worked examples provided for learners to examine algebraic reasoning in England and China. In England, these often present a problem, work through steps to solve it, and provide the "correct" solution. A number of researchers have found them to be effective for problem-solving leading to good learning outcomes in algebra [14].

4.2. Results strategies of approaching algebra

When introducing algebra, English lesson slides employed multiple forms of representation, such as pictorial images, to support the solving of word problems and to introduce key algebraic concepts. The two countries' materials differ notably in their approaches. Chinese textbooks typically introduce algebra through formal symbolic representations, using conventional algebraic notation to express abstract ideas and relationships. For example, when introducing algebraic expressions, **Table 1** shows that the Chinese textbooks present decontextualised phrases in mathematical language, rather than in word problems.

Table 1. Examples for working out algebraic expressions in the Chinese textbook

Expression questions	Answers
(1) Three times minus ;	
(2) The sum of twice and half of ;	
(3) The square of the sum of and ;	
(4) The cube root of .	

In each case, the worked solution provides an abstract, letter-symbolic form, without describing the problem-solving process or steps involved in translating the verbal phrase into algebraic notation. By contrast, the English slides introduce variables and expressions through word problems situations, including real-world problems. This aligns with the findings of Hänze & Leiss, who noted open-ended, real-world situations as a method to inject meaning and serve as authentic problems that might be met in real life [15]. In the process, several symbolic systems like words, tables and graphs are used to support algebraic understanding. For instance, both the higher level and foundational level slides provided pictorial notations (**Figure 1** and **Figure 2**) to support students in constructing meaning, recognising unknowns. Researchers have previously suggested that such multiple representations of algebraic understanding could be developed as a way to help address the difficulty of variables of notation noted at lower secondary levels [16].

In one example from the foundation level slides, students are presented with a story: "Pirate Pete and Paul have been plundering! So, each of them has a bag full of jewels." As shown below, pictorial notation was used: a bag with an unshown number of jewels in it represents the number of jewels each pirate has. The left-hand column shows an image of the result of each step, with the corresponding algebraic expression on the right. The right-hand column shows a word problem followed by its algebraic expression in the step. In order to work out and check all numbers, students must keep in mind a generic amount, the picture of the bag. According to Kaput et al., the bag and jewels notations acts as an icon to simplify at each step [10]. This scaffolds the transition from the pictorial representation to , the algebraic representation.

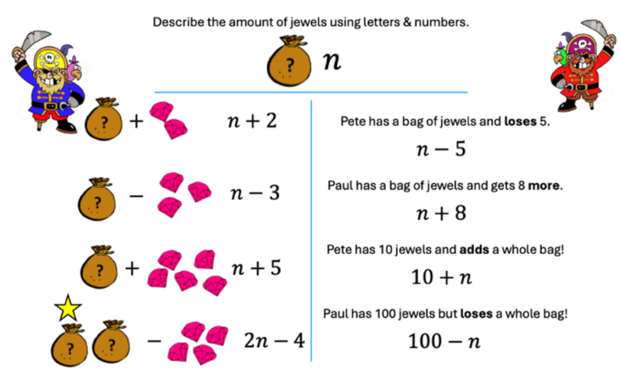

Figure 1. A worked example from the foundation level at phase 1.

Another important difference lies in the use of various contexts for problem situations in England. Contextualised problems are considered appropriate for introducing students to algebra [17]. English textbooks prefer to use story problems as the sources for solutions to problems rather than irrelevant symbols to manipulate. This seems to make it easier to give expression to such mathematical generalisations. In the English slides, a large number of examples are presented in story-problem format. This is thought to support understanding of symbols. They also use natural language terms (e.g. "many") to encourage general expressions (see **Figure 2**). For instance, a higher level example of simplifying a

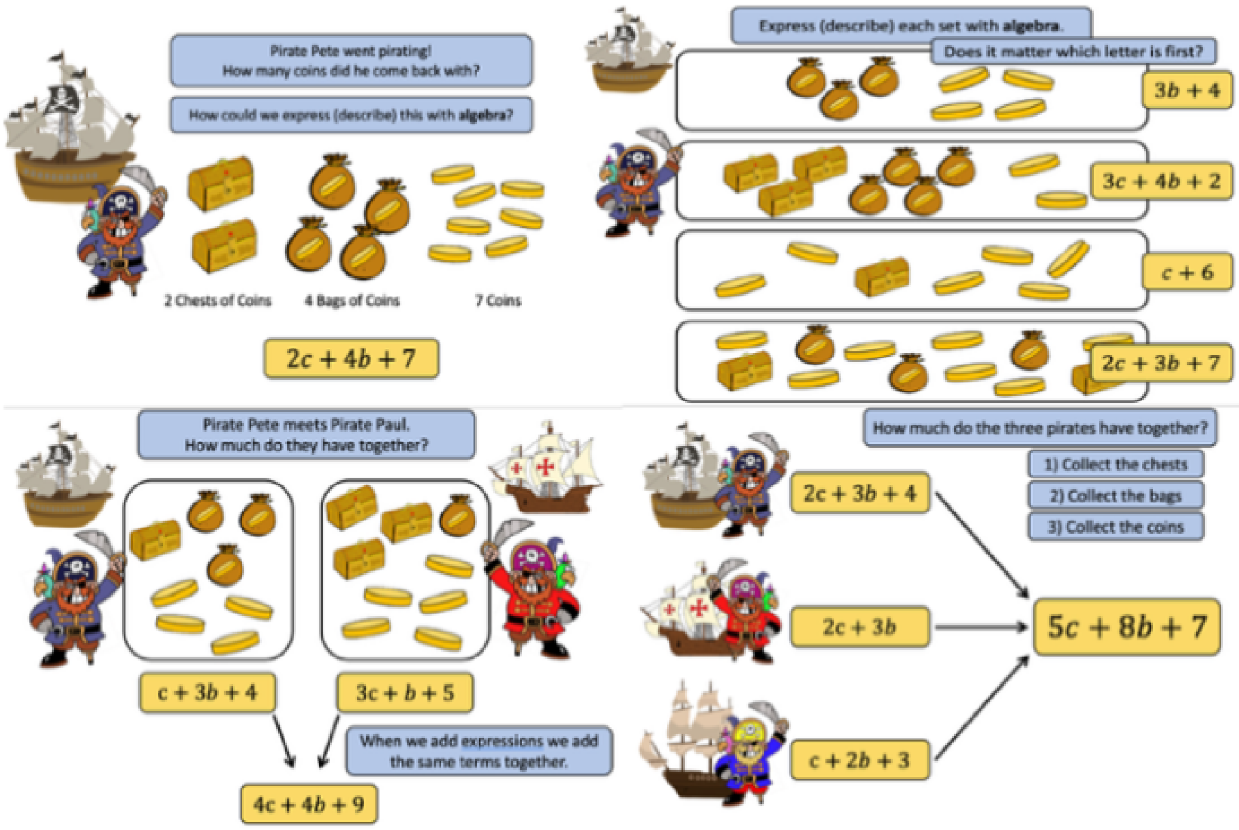

Figure 2. An example of forming the understanding of algebraic notion in England.

simple expression at Grade 7 starts with a story: "Pirate Pete went pirating, how many coins did he come back with? How could we express this with algebra?"

Through this narrative, students are introduced to algebra via a familiar and therefore meaningful real-life context (the problem), where they interact with unknown quantities of chests or bags of coins (the variables), and must assign symbols such as "b" and "c" to represent them. Through such contexts, students also build meaning for the use of algebraic properties and axioms during the manipulative process, such as when asked to collect the like terms.

5. Discussion

5.1. Ways of supporting algebraic understanding

The slides materials from England reveal links with real life contexts and extensively feature graphic representation, while the Chinese textbook provides particular opportunities for symbolisation. In addition, this study confirmed the previously reported tendency for Chinese students to show greater fluency in operations and in taking abstract approaches.

At both foundation and higher level in England, more weight was given to connecting the concept of arithmetic and algebra. The multiple representations (e.g. pictorial, tabular, graphical methods) used in word problems (**Figure 1** & **Figure 2**) reflect the importance placed on the use of algebra in the mathematical modelling of "real-life" contexts in English curricula and materials. In other words, the stress placed on everyday life problems is much greater than in China. These examples in England demonstrate the abstractness process is driven by a desire to assist students in the construction of meaning with relation to understanding.

In each case in these slides, meaning construction is assisted by invoking contexts outside the domain of algebraic manipulation, whilst the nature of that meaning is purposed in various different ways. Such intentions are targeted at helping students not only to reason around the abstractness process but also to form connections between algebra and other aspects of their experience. This process contains strong inductive structure. Therefore, the algebraic understanding in English lesson slides predominantly remains at the level of operational (process-based) understanding of algebraic thinking, whilst the examples in Chinese textbooks increasingly encourage the development of an abstract structural perspective on algebra. However, the overuse of algebraic rules and their level of complexity, combined with the difficulty of problems students face, and the level of algorithms required to manipulate algebraic expressions, may risk students falling into automatic symbolic manipulation without knowing why they do what they do [18].

5.2. Origins of these differences

Underneath the mathematical modelling of "real-life" contexts (e.g. pictorial representation of variables) in England, and the decontextualised and symbolic forms in China, the different approaches reflect contrasting views of pedagogical traditions in the two regions. The Chinese textbooks resembled an abstract, symbolic approach to algebra, placing heavy emphasis on both abstract approach and problem solving. This confirms the structural approach to algebraic understanding in Chinese textbooks.

On the other hand, the English materials stressed the importance of familiar real-world connections, which means solving algebraic problems using methods other than manual symbolic manipulation. This confirms the functional approach in England's materials. This study may suggest that materials provided to teachers in different regions signal how algebraic understanding is defined and developed. This explanation proves additional explanatory power for the different traditional methods or approaches for classrooms in England and China. However, the data from this study are insufficient to draw strong conclusions. Examining teachers' classroom practice might help us understand better how far the historical and cultural traditions of each country are stressed and to what extent the pitfalls of each are addressed.

6. Conclusion

The findings of this study provide several implications for classroom practice and curriculum development. The English materials, with their focus on contextualised and representational approaches, show the importance of providing students with opportunities to make sense of algebraic ideas through real-world situations. Such approaches appear to help learners construct meaning and develop algebraic understanding beyond symbolic manipulation.

Conversely, the Chinese textbooks demonstrate the strength of a more formal and structured progression, which supports students in developing precision, procedural fluency, and an awareness of the logical structure of algebra. However, these two orientations should not be seen as separate entities. Instead, integrating the contextual and structural approaches may offer a more balanced way of supporting students' transition from operational understanding to abstract thinking. Teachers might therefore consider designing classroom activities that combine real-world contexts with explicit attention to algebraic structure, enabling students to move flexibly between the two forms of understanding.

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