

A Study on Strategies for Developing Advanced Mathematics Reading Competence Based on Metacognitive Theory

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Abstract: Drawing on the current learning conditions and future development needs of university students, this paper explains the important role of comprehensive reading competence, of which mathematical reading competence is a key component, in developing students' capacity for self-directed learning and sustainable personal development. It also identifies the foundational role that mathematical reading competence plays in the study of Advanced Mathematics. From five perspectives, the paper discusses in detail how teachers can cultivate students' mathematical reading competence during pre-class preparation, classroom instruction, and post-class reflection, thereby continuously improving their mathematical literacy and mathematical thinking.

Keywords: Reading; Competence; Artificial intelligence; Teaching methods; Self-directed learning

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1. Introduction

In Western education, reading, writing, and arithmetic are collectively known as the classic “three Rs” and constitute the foundation of basic learning. UNESCO has explicitly identified reading, writing, and mathematics as three pillars of basic learning for all; learning in both the natural and social sciences rests on these foundations.

The reading competence expected of students should be comprehensive: grounded in general-language reading competence, it should also encompass mathematical, foreign-language, scientific, and technological reading. Comprehensive reading competence is both an important foundation and a component of self-directed learning. It is an integrated capacity that combines multiple intellectual factors and engages multiple psychological mechanisms. The capacity for self-directed learning is also an important indicator of a person's ability to achieve sustainable development. As a major component of comprehensive reading competence, mathematical reading competence refers to the ability to acquire new knowledge and skills by reading mathematical resources and actively engaging in thought on the basis of one's existing mathematical

knowledge, skills, and abilities^[1].

2. Recognizing the foundational role of Advanced Mathematics

Advanced Mathematics, offered to first-year university students as a common foundational course in engineering programs, is generally taught over one academic year. Students' attitudes toward and approaches to this course shape how they accumulate knowledge, transform their modes of thinking, and develop self-directed learning capacity throughout university. The course is broad and substantive, covering calculus, vector algebra and analytic geometry in space, infinite series, and ordinary differential equations. As one of the instrumental and foundational courses supporting quantitative analysis in subsequent subjects such as linear algebra, probability and mathematical statistics, statistics, and operations research, it provides essential support for students' university studies and future careers.

Teaching practice has shown that students with limited proficiency in mathematical language are less sensitive to mathematical information in class, shift between modes of thought more slowly, and assimilate knowledge less effectively. Their mathematical comprehension is correspondingly weaker, and they frequently encounter difficulties and make errors when interpreting problems.

Because mathematical language is highly abstract and precise, because its meanings may shift rapidly with context, and because mathematical materials are organized with rigorous logic, mathematical reading can cultivate abstract thinking, logical reasoning and proof, and creative thinking. It can also help students concentrate, broaden their perspectives, reason rigorously, and think flexibly.

3. Recognizing the educational functions of mathematical reading

"Teaching mathematics is, in essence, teaching mathematical language," and language cannot be learned without reading.

Teachers should incorporate mathematical reading into the core stages of Advanced Mathematics instruction. Students are the subjects and central participants of learning. A teacher's explanations can become part of students' knowledge structures and improve their cognitive capacity only when those explanations arouse cognitive conflict, create tension between prior and new knowledge, and prompt conscious, active reflection.

Consider the definition of a limit. A verbal definition is first given: as the independent variable x follows a particular pattern of change, what pattern does the function $f(x)$ follow? To describe these two processes precisely, the limit is then expressed in mathematical language through the epsilon-delta (ϵ - δ) definition. Students can usually understand the verbal description by reading it independently, but they find the six different forms of the ϵ - δ definition extremely difficult. The teacher needs only to explain the most commonly used form clearly and can leave the other five forms for students to read on their own. This approach uses class time efficiently, actively engages students in reading, and gradually cultivates mathematical ways of thinking and rigorous reasoning.

Teachers must therefore change their assumptions and move beyond the one-way model in which the teacher speaks and students listen. They should avoid lecturing throughout the entire class, reject the view that teaching is solely the transmission of knowledge rather than the cultivation of methods and abilities, and correct the belief that students with weak foundations are unsuitable for competence-oriented education.

When students read textbooks, handouts, and reference works to identify, pose, and attempt to solve problems, they develop divergent thinking, an essential component of creative thought. Guiding students' reading is also a primary means through which teachers exercise their instructional leadership and shift the teaching relationship from passive reception to active interaction ^[2].

4. Framing learning as problem posing and problem solving

To cultivate innovative talent, educators must use problems as the organizing thread for developing creative thinking and encourage students to identify, pose, and solve problems. Through pre-class reading, students should gain an initial understanding of the basic content and logic of the teaching materials. If they discover that their command of prior knowledge connected to the new lesson is inadequate, they should promptly consult relevant sources and review that knowledge, thereby laying a sound foundation for new learning.

The mean value theorems of differential calculus are both important and difficult for students. When teaching Rolle's theorem, Lagrange's mean value theorem, Cauchy's mean value theorem, and Taylor's mean value theorem, teachers naturally require students to master the hypotheses and conclusions of each theorem and apply them to specific problems. More importantly, students should be guided to identify and resolve the distinctions and relationships among the four theorems through reading, independent study, and reflection. Guided inquiry and comparison deepen students' understanding and help them master and apply the theorems flexibly.

By reading textbooks and handouts carefully, students identify problems and formulate questions. They then attend class with unresolved questions in mind, improving the efficiency of problem solving. Teachers need not answer every difficult question exhaustively; an appropriately limited prompt can activate students' thinking more effectively.

After explaining a difficult or important topic, a teacher should set aside time for students to read the relevant material and provide immediate feedback, allowing the pace of instruction to be adjusted. Students should also be encouraged to reread the textbook material after class so that they can identify remaining uncertainties, seek help proactively, and genuinely master the new knowledge. At the end of a unit, teachers can organize systematic review and guide students in writing a unit summary. In this way, students clarify the structure of knowledge, identify reading priorities, connect new material with what precedes and follows it, and develop their abilities to summarize, observe, comprehend, associate, and compare.

Students, especially those who are struggling, should be enabled to experience that they have learned something through reading, which strengthens their willingness to read mathematics independently. The central aim of competence-oriented education is to enable every student to develop fully. Collective instruction alone cannot achieve this aim; an effective approach combines collective instruction with individual learning, and effective individual learning depends on teaching students how to read. Teachers should also encourage students to read works on the history of mathematics so that they understand the background of mathematical knowledge, the systematic structure of mathematical science, and the developmental directions of its branches. Such reading supports both broader knowledge and the study of fundamentals. Teachers can recommend materials that display creativity while accurately reflecting advanced achievements in contemporary science, thereby giving students some familiarity with frontier knowledge. Through reading, students can move beyond convergent thinking and develop divergent thinking ^[3].

As teaching reform progresses, assessment and evaluation are shifting from single measures to more comprehensive approaches. Students can acquire a rich foundation in mathematics through reading. They may use online multimedia courseware or lecture videos from other countries to study Advanced Mathematics and prepare for subsequent courses. In an environment shaped by Internet Plus and AI-enabled learning, students need strong reading competence to study independently, gather information efficiently, and engage in lifelong learning. If a student's field of study does not align with the requirements of an initial position, only continued self-directed learning and professional growth can reconcile the demands of that position with future development and enable the student to meet society's need for highly qualified professionals.

5. Applying classroom reading guidance strategies to improve teaching efficiency

In class, teachers should deliberately revisit and reread key content, while asking students to summarize material they can understand through independent reading. Teachers should focus on the origins and background of problems, the ideas and methods used to solve them, the key and difficult points, and their own insights into the teaching materials, with particular emphasis on analytical approaches and solution methods. Students who find it difficult to sustain attention may become more mentally engaged when solving problems independently. Their thinking is freed from rigid constraints, enabling them to exercise their analytical and problem-solving abilities and overcome the tendency to reproduce existing methods without innovating.

For example, students have already encountered derivatives and differentials in secondary school and have learned certain basic formulas, rules, and theorems. University teachers can therefore concentrate on showing students how to derive selected differentiation formulas from the definition of the derivative, how to use that definition to solve derivative problems involving piecewise functions, and how to extend it to differential calculus for functions of several variables. Organizing classroom reading does not mean leaving students without direction. The teacher must act as coordinator and manager, guiding the reading process and maintaining command of the instructional setting.

In selecting mathematical reading materials, teachers should cover foundational classics while also introducing the history of the discipline and problems at the research frontier, thereby creating an open system of knowledge. When guiding reading, teachers can demonstrate their own reading practices and consider questions from students' perspectives, so that the thinking of teachers and students becomes integrated. Depending on the circumstances, classroom instruction can employ three modes: overview reading, question-driven reading, and close reading.

The textbook is the basis of instruction and must be read closely, repeatedly, and critically. Teachers should take the lead in mathematical reading. Textbooks are carefully written by experts who consider students' psychological and physiological characteristics, principles of education and instruction, and the distinctive features of mathematics; consequently, they have substantial reading value. Some teachers, however, do not make full use of textbooks and treat them merely as lecture notes or collections of exercises. This helps explain why some students continue to learn poorly even when classroom explanations are excellent ^[4].

Infinite series provide a useful example. In some problems, students must determine the sum function to

which a power series converges; in others, they must work in the opposite direction and find the power-series expansion of a function. These procedures serve different practical needs. When guiding students through this material, teachers should make the relationship between the two tasks explicit and thereby resolve students' confusion.

6. Integrating new teaching models and technologies

Modern teaching methods, computer resources, and advanced communication technologies can complement traditional classroom instruction, improve the efficiency and quality of information delivery, and steadily enhance learning outcomes. The deep integration of artificial intelligence is shifting digital education from the provision of resources toward precision education. AI enriches classroom presentation through multimodal materials, supports differentiated instruction through large-scale learning data, reduces teachers' administrative burden through intelligent assistants, and magnifies the instructional value of digital hardware. In advancing competence-oriented education, teachers should innovate in both content and pedagogy. They can use AI to distribute tiered reading resources and stimulate deeper thinking around difficult and important course content; use AI assistance to refine writing frameworks and improve logical expression while preserving training in original writing; and use mathematical simulation and intelligent analysis of errors to strengthen mathematical reasoning. By using AI responsibly and adopting human-AI collaboration, educators can cultivate students' reading foundations, written expression, and mathematical and logical literacy, preparing highly qualified professionals for the intelligent era and advancing the practical implementation of talent-development strategies ^[5-7].

Massive open online courses, XuetangX, microlectures, and flipped classrooms collectively create semi-structured, topic-based microenvironments in which learning resources are organized and presented. Communication platforms such as WeChat groups, QQ groups, and Rain Classroom also enable teachers to monitor students' learning progress, classroom responses, and completion of after-class assignments in a timely manner. The resulting interaction, discussion, and question-and-answer activities allow students to improve their mathematical literacy continuously through reading about Advanced Mathematics.

7. Conclusion

In an era of rapid change and continuing social transformation, the breakthrough development and widespread adoption of AI have fundamentally reshaped the dissemination of knowledge and the structure of society's demand for talent. These developments also present teachers with new responsibilities and challenges. As AI empowers industries across the economy, self-directed learning has become a core capacity for individuals seeking to establish themselves and continue to grow. AI can provide students with abundant learning resources, intelligent question-answering support, and personalized learning plans, but it remains an auxiliary tool. Only students with a strong awareness of and capacity for self-directed learning can use AI appropriately to identify and remedy gaps and broaden their understanding, rather than becoming dependent on technology and intellectually passive. By mastering independent inquiry, active knowledge seeking, and continuous improvement, students can keep pace with technological and social change, respond confidently to future occupational transformations and unforeseen challenges, and meet the developmental needs of a digital society ^[8,9].

Disclosure statement

The author declares no conflict of interest.

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